Exercise 33

For what values of x does the graph of f have a horizontal tangent?

$$f(x) = x + 2\sin x$$

Solution

The graph of f has a horizontal tangent wherever the first derivative is zero. Calculate the first derivative.

$$f'(x) = \frac{d}{dx}[f(x)]$$

$$= \frac{d}{dx}(x + 2\sin x)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(2\sin x)$$

$$= (1) + (2\cos x)$$

$$= 1 + 2\cos x$$

Set it equal to zero.

$$1 + 2\cos x = 0$$

Solve for x.

$$\cos x = -\frac{1}{2}$$

$$x = \left\{ \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi \right\}$$

Here n is an integer.